Developments in Geomathematics 3

INTRODUCTION TO THE MATHEMATICS OF INVERSION IN REMOTE SENSING AND INDIRECT MEASUREMENTS

S. TWOMEY

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