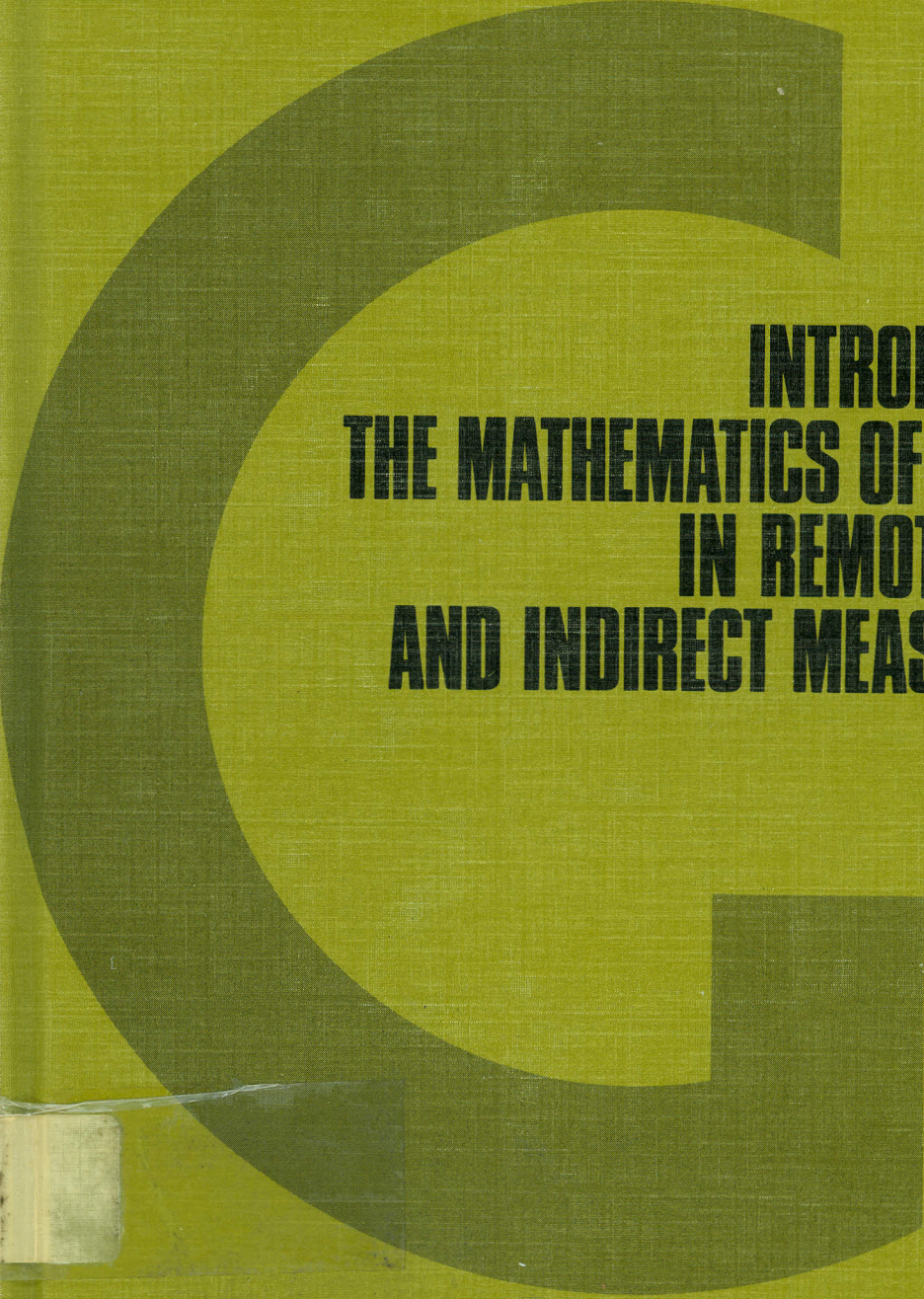


Developments in Geomathematics 3



**INTRODUCTION TO
THE MATHEMATICS OF INVERSION
IN REMOTE SENSING
AND INDIRECT MEASUREMENTS**

S. TWOMEY

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